A NOTE ON PROBABILITY AND SEVERITY OF RUIN FOR GENERALIZED LINDLEY CLAIM SIZE DISTRIBUTION

NOTA SOBRE LA PROBABILIDAD Y SEVERIDAD DE RUINA CUANDO LA CANTIDAD RECLAMADA SIGUE LA DISTRIBUCIÓN GENERALIZADA LINDLEY

Emilio Gómez–Déniz[1], M. E. Ghitany[2], D. K. Al–Mutairi[3]

Abstract

It is well known that, in a classical continuous time surplus process with given insurer's initial surplus, closed-form expressions for the probability and severity of ruin (the probability that ruin occurs and that the insurer's deficit at the time of ruin is less than specified value), exist for few claim size distributions such as the exponential and mixed exponential distributions. This paper provides a closed-form expressions for the probability and severity of ruin in the case where the claim size follows the generalized Lindley distribution which contains as special cases the exponential and the Lindley distributions. An upper bound for the probability of ruin is obtained via Lundberg's inequality. Numerical applications are shown to illustrate the results.

Keywords: Exponential distribution, generalized Lindley distribution, Lundberg's inequality, ruin function, severity of ruin.

[1] Department of Quantitative Methods in Economics and T*i* DES Institute. University of Las Palmas de Gran Canaria, Spain. emilio.gomez-deniz@ulpgc.es (author of correspondence).
[2] Department of Statistics and Operations Research, Faculty of Science, Kuwait University, Kuwait. meghitany@yahoo.com
[3] Department of Statistics and Operations Research, Faculty of Science, Kuwait University, Kuwait. mutairid@yahoo.com
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Resumen

Resulta bien conocido que en el modelo clásico de ruina solamente existen expresiones cerradas para la función de ruina para el caso en que la cuantía de las reclamaciones sigan una distribución exponencial o una suma convexa de la misma. En este trabajo se proporcionan funciones de ruina explícitas para el caso en que la cantidad reclamada siga una distribución generalizada Lindley así como para el caso particular de la misma que da lugar a la distribución Lindley. Se proporcionan también una cota superior para la probabilidad de ruina obtenida utilizando la desigualdad de Lundberg. Los resultados se muestran con aplicaciones numéricas.

Palabras Clave: Distribución exponencial, Distribución generalizada Lindley, Desigualdad de Lundberg, Función de ruina, Severidad de la ruina.

1 Introduction

It is well–known that in a classical continuous time surplus process when the insurer’s initial surplus is given, an explicit expression for the probability of ultimate ruin exists for few claim size distributions such as the exponential and mixed exponential distributions. When the claim size distribution is exponential simple analytic results for the ruin probability in infinite time may be possible. Nevertheless, Grandell (1990) points out that there is really no reason to believe that the exponential distribution is a realistic description of the claim behavior. On the other hand, the class of mixed exponential distributions is somewhat limited because the mode of such a distribution is necessarily at 0 (see Gerber *et al.* (1987)). Therefore, some efforts have been made in statistical literature in order to find other probabilistic models to be applied in this setting. Although for most of the general claim size distributions, e.g. heavy–tailed, the Laplace transform technique does not work, explicit expression under other assumptions, such as Pareto distributions, have been obtained but they are too complicated and require large computation to obtain the values of the ultimate ruin probability. For example, Garcia (2005) derived complicated exact solutions under series representation and Seal (1980), Makov (2001) and Wei and Yang (2004) under integral representations. Grandell and Segerdahl (1971) show that for the gamma claim amount distribution under some restrictions on the parameters the exact value of ruin probability can be computed via a formula which involves a complicated integral. In Ramsay (2003), an expression based on numerical integration was derived for the probability of ultimate ruin under the classical compound Poisson risk model, given an initial reserve of u in the case of Pareto individual claim amount distributions. Furthermore, Albrecher *et al.* (2011) have obtained closed–form expressions for ruin probability functions under some kind of dependence assumption. Also, explicit expressions for the infinite-time ruin probability exist for the (large) class of phase-type distributions (see Asmussen, 2000).

This paper is concerned with the computation of the probability of ruin and with the severity of ruin when the claim size distribution follows a generalized Lindley distribution. The generalized Lindley distribution is a two parameters distribution proposed recently by Zakerzadeh and Dolati (2009) and which generalizes in an easy way the exponential distribution and the Lindley distribution. Its probability density function (p.d.f.) is given by

$$f(x) = \frac{1}{\lambda(1+\gamma\lambda)}(1+\gamma x)\exp\left(-\frac{x}{\lambda}\right), \quad x > 0,\ \lambda > 0,\ \gamma \geq 0.$$

When $\gamma = 0$ the p.d.f. (1) reduces to the exponential distribution with parameter $\lambda > 0$. Also, when $\gamma = 1$ the p.d.f. (1) reduces to the Lindley distribution with parameter $\lambda > 0$, see Lindley (1958) and Ghitany *et al.* (2008).

Sankaran (1971) introduced the discrete Poisson-Lindley distribution by assigning the Lindley distribution to the Poisson parameter and applied his model to several discrete data sets in different fields of applications. The moments and maximum likelihoo Estimation methods for the discrete Poisson-Lindley distribution are studied by Ghitany and Al-Mutairi (2009). Gómez–Déniz *et al.* (2012) introduced a discrete Lindley distribution which can be considered as a competitive model compared to those ones traditionally used in actuarial statistics when both number of claims and size of a single claim must be analysed in the context of collective risk model.

Based on the probability density function (1) we obtain a closed-form expression for the ruin probability function, $\psi(u)$, and also a closed-form expression for the upper bound of the ruin probabilities. Furthermore, the probability that ruin occurs and that the insurer's deficit at ruin is at most y is obtained also in closed-form expression. The results contains as particular cases the exponential claim and also the Lindley claim. Numerical applications are provided to illustrate the result.

The rest of the paper is organized as follows. In Section 2 we present a closed-form expression for the ruin probability function when the claim size follows the generalized Lindley distribution. In addition, upper bounds for the probability of the ruin function are also provided. Section 3 is concerned with the calculating the severity of ruin (probability that ruin occurs and that the insurer's deficit at ruin is less than a specified value). Finally, numerical applications appear in Section 4, where comparisons with the exponential distribution are shown.

2 Probability of ruin

In the classical risk model, the ruin probability function focus mainly on the claim number process (usually a Poisson process) and the claim size distributions. It is assumed that the claims arrival process $\{N(t)\}_{t\geq 0}$, where $N(0)=0$, is a homogeneous Poisson process with Poisson parameter $\alpha > 0$. The claim costs are assumed to be independent and identically distributed non–negative random variables with known cumulative distribution function (c.d.f.) $F(x)$ with finite mean μ, and independent of $\{N(t)\}_{t\geq 0}$.

The surplus process of an insurance portfolio is defined as the wealth obtained by the premium payments minus the reimbursements made at the times of claims. When this process becomes negative (if ever), we say that ruin has occurred. Let $\{U(t)\}_{t\geq 0}$ be a classical continuous time surplus process, the surplus process at time t given the initial surplus $u=U(0)$, the dynamic of $\{U(t)\}_{t\geq 0}$ is given by

$$U(t)=u+ct-S(t),$$

where $S(t)=\sum_{i=1}^{N(t)} X_i$ is the aggregate claim amount up to time t and $S(t)=0$ if $N(t)=0$. Here, $u\geq 0$ is the insurer's initial risk surplus at $t=0$ and $c=(1+\theta)\alpha\mu$ is the insurer's rate of premium income per unit time with loading factor $\theta \geq 0$.

The probability of ultimate ruin, $\psi(u)$, given an initial surplus $u\geq 0$, is defined as

$$\psi(u)=\Pr\left[U(t)<0 \text{ for some } t>0 \,|\, U(0)=u\right].$$

Note that $\psi(u)$ is small for sufficiently large u.

It is well known, see for example Gerber (1979), Rolski *et al.* (1999) and Dickson (2005), that the general solution to $\psi(u)$ satisfies the following Volterra integral equation,

$$\psi(u) = \frac{1}{1+\theta}\left[K(u) + \int_0^u \psi(u-x)H(x)dx\right],$$

where

$$H(x) = \frac{1-F(x)}{\mu},$$
$$K(u) = \int_u^\infty H(x)dx.$$

For $z > 0$, consider the following Laplace transforms:

$$\psi^*(z) = \int_0^\infty e^{-zu}\psi(u)du,$$
$$H^*(z) = \int_0^\infty e^{-zx}H(x)dx,$$
$$K^*(z) = \int_0^\infty e^{-zu}K(u)du = \frac{1}{z}\left[1-H^*(z)\right].$$

Ramsay (2003) showed that the Laplace transform of $\psi(u)$ can be expressed as follows

$$\psi^*(z) = \frac{K^*(z)}{1+\theta-H^*(z)}.$$

When $F(x) = 1-\exp(x/\lambda)$, i.e. the claim size follows an exponential distribution with parameter $\lambda > 0$, it is possible to invert the Laplace transform given in (2) and thus to obtain a closed-form expression for $\psi(u)$ (see Gerber (1979) and Dickson (2005)), specifically,

$$\psi(u) = \frac{1}{1+\theta}\exp\left[-\frac{\theta u}{\lambda(1+\theta)}\right], \quad u \geq 0.$$

Expression (3) is simple and is the unique explicit formula existing in the literature (see Dickson (2005) and Gerber (1979). All other explicit ruin probability functions are based on using a finite mixture of exponential distributions. Furthermore, for the gamma and Pareto distributions closed-form expressions exist for the ruin probability function but they are too complicated to compute the values of the ultimate ruin probability. In this sense see Ramsay (2003).

Now consider the generalized Lindley distribution with p.d.f. (1). This distribution has c.d.f.

$$F(x) = 1 - \frac{1+\gamma(\lambda+x)}{1+\gamma\lambda}\exp\left(-\frac{x}{\lambda}\right),$$

moment generating function (m.g.f.)

$$M(r) = E(e^{rX}) = \frac{1+\lambda(\gamma-r)}{(1+\gamma\lambda)(1-\lambda r)^2},$$

and mean

$$\mu = E(X) = \frac{\lambda(1+2\gamma\lambda)}{1+\gamma\lambda}. \qquad (6)$$

Now, in this case, it is straightforward to show that function (1) $H(x)$ and $K(u)$ are given by

$$H(x) = \frac{1+\gamma(\lambda+x)}{\lambda(1+2\gamma\lambda)}\exp\left(-\frac{x}{\lambda}\right),$$

$$K(u) = \frac{1+\gamma(2\lambda+u)}{1+2\gamma\lambda}\exp\left(-\frac{u}{\lambda}\right).$$

Therefore, we obtain

$$H^*(z) = \frac{1+\lambda[2\gamma+(1+\gamma\lambda)z]}{(1+2\gamma\lambda)(1+\lambda z)^2},$$

$$K^*(z) = \frac{\lambda\{1+\lambda[3\gamma+(1+2\gamma\lambda)z]\}}{(1+2\gamma\lambda)(1+\lambda z)^2}.$$

Finally, using (7) and (8) in (2), the Laplace transform of $\psi(u)$ is given by

$$\psi^*(z) = \frac{\lambda\{1+\lambda[3\gamma+(1+2\gamma\lambda)z]\}}{\theta(1+2\gamma\lambda)(1+\lambda z)^2+\lambda z\{1+\lambda[3\gamma+(1+2\gamma\lambda)z]\}}.$$

The inverse of the Laplace transform of (9) gives the following closed-form expression for the ruin probability function when the claim size follows the generalized Lindley distribution

$$\psi(u) = \frac{1}{2(1+\theta)r_1}\left\{(r_1-r_3)\exp(-R_1u)+(r_1+r_3)\exp(-R_2u)\right\}, \quad u \geq 0,$$

where

$$r_1 = \sqrt{\gamma\lambda[\gamma\lambda(9+8\theta)+2\theta+3]+r_3},$$
$$r_2 = 2\theta(1+\gamma\lambda)+r_3,$$
$$r_3 = 1+\gamma\lambda(3+2\theta).$$
$$R_1 = \frac{r_1+r_2}{2\mu(1+\theta)(1+\gamma\lambda)},$$
$$R_2 = \frac{r_2-r_1}{2\mu(1+\theta)(1+\gamma\lambda)}.$$

it can be checked that when $\gamma = 0$, i.e. exponential claim size distribution, expression (10) reduces to expression (3).

Expression (10), for generalized Lindley claim size, depends on three parameters given by θ, γ and λ while the corresponding expression for a two-component mixture of exponentials claim size, depends on four parameters. For more details, see Panjer and Willmot (1992).

It is well known that Lundberg's inequality is given by

$$\psi(u) \le \exp(-Ru),$$

where R is the adjustment coefficient which is the smallest positive solution in r of the equation

$$\alpha + c\,r - \alpha M(r) = 0,$$

in $(0,\kappa)$, provided that $\lim_{r\to\kappa^-} M(r) = \infty$, where $M(r)$ is the m.g.f. of the claim size distribution, see Dickson (2005, chapter 7). For the generalized Lindley claim size distribution with m.g.f. (5), equation (13) is equivalent to

$$\mu(1+\theta)(1+\gamma\lambda)\lambda^2 r^2 - [2\mu(\theta+1)-\lambda](1+\gamma\lambda)\lambda r \\ + \mu(1+\theta)(1+\gamma\lambda)-(1+2\gamma\lambda)\lambda = 0, \qquad r \in (0,1/\lambda).$$

The unique solution of the quadratic equation (14) is given by

$$R = \frac{1+2\theta+\gamma\lambda(4\theta+3)-r_1}{2\mu(1+\theta)(1+\gamma\lambda)},$$

where r_1 is given by (11). Note that when $\gamma = 0$, i.e. exponential claim size distribution, we have $R = \theta/[\lambda(1+\theta)]$.

3 Severity of ruin

Actuaries deal with ruin theory are also interested in the severity of the insurer's deficit at the time of ruin should ruin occur. The time of ruin, given an initial surplus u, is defined by

$$T_u = \inf_{t>0} \{t : U(t) < 0 \,|\, U(0) = u\},$$

with the convention that $T_u = \infty$, if ruin does not occur.

It follows that

$$G(u, y) = \Pr[T_u < \infty, -y < U(T_u) < 0], \qquad u, y \geq 0,$$

is the probability that ruin occurs and that the deficit at the time of ruin is less than y. It can be seen (see Gerber *et al.* (1987), Dickson and Waters (1992) and Dickson (2005)) that the Laplace transform of $G(u, y)$ is given by

$$G^*(z, y) = \frac{\psi(0)\eta^*(z, y)}{1-\psi(0)H^*(z)}, \qquad z, y \geq 0,$$

where $\psi(u)$, $H^*(z)$ are defined in the previous section and

$$\eta^*(z, y) = \int_0^\infty e^{-zu} \int_u^{u+y} H(x)dxdu.$$

For claim size distribution following the generalized Lindley distribution considered in this paper, we have $\psi(0) = 1/(1+\theta)$, $H^*(z)$ is given by (7) and

$$\eta^*(z, y) = \frac{\lambda}{(1+2\gamma\lambda)(1+\lambda z)^2} \Big[1 + \lambda(3\gamma + (1+2\gamma\lambda)z) \\ -(1+\lambda z + \gamma(y(1+\lambda z) + \lambda(3+2\lambda z)))\exp\left(-\frac{y}{\lambda}\right)\Big].$$

It follows that

$$G^*(z, y) = \frac{\lambda h_1(z, y)}{h_2(z, y)},$$

where

$$h_1(z, y) = 1 + \lambda(3\gamma + (1+2\gamma\lambda)z) \\ -(1+\lambda z + \gamma(\lambda(3+2\lambda z) + y(1+\lambda z))\exp\left(-\frac{y}{\lambda}\right),$$

$$h_2(z,y)=\theta(1+2\gamma\lambda)(1+\lambda z)^2+\lambda z(1+\lambda(3\gamma+(1+2\gamma\lambda)z).$$

Now, inverting the last Laplace transform, we obtain the probability that ruin occurs and that the insurer's deficit at ruin is at most y :

$$G(u,y)=\psi(u)\left[1-\frac{r_4(y)-r_5(y)\exp(R_3u)}{(1+2\gamma\lambda)\left(r_3-r_1-(r_1+r_3)\exp(R_3u)\right)}\exp\left(-\frac{y}{\lambda}\right)\right],$$

where

$$R_3=\frac{r_1}{\lambda(1+\theta)(1+2\gamma\lambda)},$$
$$r_4(y)=(1+2\gamma\lambda)(r_3-r_1)+\gamma y(1+\gamma\lambda-r_1),$$
$$r_5(y)=(1+2\gamma\lambda)(r_1+r_3)+\gamma y(1+\gamma\lambda+r_1).$$

Note that $\lim_{y\to\infty}G(u,y)=\psi(u)$ is the ultimate ruin probability.

4 Numerical illustrations

In this section we illustrate numerically the usefulness of (10), (12) and (17) obtained in this paper. The numerical findings can be summarized as follows:

1. Tables 0 and 1 show the ruin probabilities for the exponential, Lindley and generalized Lindley distributions for selected values of parameters. The results show that the ruin probabilities are larger for the Lindley and the generalized Lindley distribution than the ones obtained with the exponential distribution, i.e. the ruin probabilities increase as γ increases.

2. Tables 2 and 3 show the upper bounds of the ruin probabilities appearing in Tables 0 and 1 based on (12), where R is given by (15). Again, the upper bounds of the ruin probabilities are larger for the Lindley and the generalized Lindley distribution than the ones obtained with the exponential distribution, i.e. the upper bounds of the ruin probabilities increase as γ increases.

3. Table 4 shows the probabilities of severity of ruin were computed and the results appear in . The results show that, for fixed u,θ,γ,

probabilities of severity of ruin increase as y increases. For fixed y,θ,γ, probabilities of severity of ruin decrease as u increases.

Table 1: Ruin probabilities for several premium loading factors θ.

Exponential distribution, $\lambda=1,\gamma=0$

u	$\theta=0.10$	$\theta=0.25$	$\theta=0.50$	$\theta=0.75$	$\theta=1.00$
1	0.830092	0.654985	0.477688	0.372251	0.303265
2	0.757957	0.536256	0.342278	0.242499	0.183940
3	0.692091	0.439049	0.245253	0.157973	0.111565
4	0.631949	0.359463	0.175731	0.102910	0.067667
5	0.577033	0.294304	0.125917	0.067039	0.041042
6	0.526889	0.240955	0.090223	0.043672	0.024893
7	0.481103	0.197278	0.064648	0.028449	0.015098
8	0.439296	0.161517	0.046322	0.018533	0.009157
9	0.401121	0.132239	0.033191	0.012073	0.005554
10	0.366264	0.108268	0.023782	0.007865	0.003368

Lindley distribution, $\lambda=1,\gamma=1$

u	$\theta=0.10$	$\theta=0.25$	$\theta=0.50$	$\theta=0.75$	$\theta=1.00$
1	0.852684	0.694733	0.526794	0.422020	0.350975
2	0.797182	0.598732	0.410566	0.305867	0.240771
3	0.744607	0.514780	0.318460	0.220124	0.163665
4	0.695317	0.442273	0.246604	0.157991	0.110838
5	0.649241	0.379892	0.190849	0.113279	0.074946
6	0.606206	0.326285	0.147669	0.081188	0.050645
7	0.566020	0.280237	0.114251	0.058179	0.034214
8	0.528498	0.240686	0.088392	0.041689	0.023112
9	0.493462	0.206716	0.068386	0.029872	0.015611
10	0.460749	0.177541	0.052907	0.021404	0.010545

Table 2: Ruin probabilities for several premium loading factors θ.

GLD distribution, $\lambda=1,\gamma=0.5$

u	$\theta = 0.10$	$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$	$\theta = 1.00$
1	0.847486	0.685490	0.515228	0.410192	0.339561
2	0.788508	0.584657	0.394846	0.291066	0.227359
3	0.733190	0.497875	0.301626	0.205556	0.151291
4	0.681625	0.423749	0.230134	0.144881	0.100397
5	0.633650	0.360594	0.175505	0.102031	0.066541
6	0.589041	0.306832	0.133819	0.071829	0.044078
7	0.547569	0.261080	0.102028	0.050560	0.029191
8	0.509017	0.222149	0.077787	0.035587	0.019329
9	0.473178	0.189022	0.059305	0.025047	0.012799
10	0.439863	0.160835	0.045214	0.017629	0.008474

GLD distribution, $\lambda = 1, \gamma = 2$

u	$\theta = 0.10$	$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$	$\theta = 1.00$
1	0.856646	0.701822	0.535730	0.431206	0.359873
2	0.803661	0.609352	0.422566	0.317256	0.251152
3	0.753070	0.527485	0.331311	0.231362	0.173280
4	0.705443	0.456219	0.259252	0.168190	0.119030
5	0.660774	0.394480	0.202734	0.122128	0.081626
6	0.618920	0.341071	0.158503	0.088643	0.055939
7	0.579713	0.294887	0.123914	0.064330	0.038326
8	0.542989	0.254955	0.096870	0.046683	0.026256
9	0.508591	0.220430	0.075728	0.033876	0.017986
10	0.476372	0.190580	0.059200	0.024583	0.012321

Table 3: Upper bounds for ruin probabilities for several premium loading factors θ

Exponential distribution, $\lambda = 1, \gamma = 0$

u	$\theta = 0.10$	$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$	$\theta = 1.00$
1	0.913101	0.818731	0.716531	0.651439	0.606531
2	0.833753	0.670320	0.513417	0.424373	0.367879
3	0.761300	0.548812	0.367879	0.276453	0.223130
4	0.695144	0.449329	0.263597	0.180092	0.135335
5	0.634736	0.367879	0.188876	0.117319	0.082085
6	0.579578	0.301194	0.135335	0.076426	0.049787
7	0.529213	0.246597	0.096972	0.049787	0.030197
8	0.483225	0.201897	0.069483	0.032433	0.018315
9	0.441233	0.165299	0.049787	0.021128	0.011109
10	0.402890	0.135335	0.035674	0.013763	0.006737

Lindley distribution, $\lambda = 1, \gamma = 1$

u	$\theta = 0.10$	$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$	$\theta = 1.00$
1	0.933707	0.858862	0.773662	0.716531	0.675451
2	0.871809	0.737644	0.598553	0.513417	0.456234
3	0.814014	0.633535	0.463077	0.367879	0.308164
4	0.760050	0.544119	0.358265	0.263597	0.208149
5	0.709664	0.467323	0.277176	0.188876	0.140595
6	0.662618	0.401366	0.214441	0.135335	0.094964
7	0.618691	0.344718	0.165904	0.096972	0.064144
8	0.577676	0.296066	0.128354	0.069483	0.043320
9	0.539380	0.254279	0.099302	0.049787	0.029264
10	0.503623	0.218391	0.076826	0.035674	0.019766

Table 4: Upper bounds for ruin probabilities for several premium loading factors θ

GLD distribution, $\lambda = 1, \gamma = 0.5$

u	$\theta = 0.10$	$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$	$\theta = 1.00$
1	0.929592	0.850881	0.762398	0.703821	0.662122
2	0.864142	0.723998	0.581251	0.495364	0.438406
3	0.803299	0.616036	0.443145	0.348648	0.290279
4	0.746741	0.524173	0.337853	0.245385	0.192200
5	0.694165	0.446008	0.257578	0.172707	0.127260
6	0.645290	0.379500	0.196377	0.121555	0.084261
7	0.599857	0.322909	0.149717	0.085553	0.055791
8	0.557622	0.274757	0.114144	0.060214	0.036940
9	0.518361	0.233785	0.087023	0.042379	0.024459
10	0.481865	0.198923	0.066346	0.029827	0.016195

GLD distribution, $\lambda = 1, \gamma = 2$

u	$\theta = 0.10$	$\theta = 0.25$	$\theta = 0.50$	$\theta = 0.75$	$\theta = 1.00$
1	0.936651	0.864583	0.781746	0.725659	0.685024
2	0.877315	0.747503	0.611127	0.526581	0.469258
3	0.821738	0.646278	0.477746	0.382119	0.321453
4	0.769682	0.558761	0.373476	0.277288	0.220203
5	0.720923	0.483095	0.291964	0.201217	0.150845
6	0.675253	0.417676	0.228242	0.146015	0.103332
7	0.632477	0.361115	0.178427	0.105957	0.070785
8	0.592410	0.312214	0.139485	0.076888	0.048489
9	0.554881	0.269935	0.109042	0.055795	0.033216
10	0.519730	0.233381	0.085242	0.040488	0.022754

Table 5: Probabilities of severity of ruin with a premium loading factor, $\theta = 0.25$

	y	$\gamma = 0$	$\gamma = 1$	$\gamma = 0.5$	$\gamma = 2$
$u = 0$	1	0.505696	0.407595	0.432121	0.387975
	3	0.760170	0.720341	0.730298	0.712375
	5	0.794610	0.785626	0.787872	0.783829
	∞	0.800000	0.800000	0.800000	0.800000
$u = 5$	1	0.186035	0.206660	0.202357	0.209781
	3	0.279651	0.347386	0.332254	0.358771
	5	0.292321	0.374266	0.355821	0.388198
	∞	0.294304	0.379892	0.360594	0.394480
$u = 10$	1	0.068438	0.096599	0.090272	0.101367
	3	0.102878	0.162357	0.148202	0.173336
	5	0.107539	0.174913	0.158708	0.187546
	∞	0.108268	0.177541	0.160835	0.190580
$u = 15$	1	0.025177	0.045143	0.040262	0.048970
	3	0.037846	0.075873	0.066099	0.083737
	5	0.039561	0.081741	0.070785	0.090602
	∞	0.039829	0.082968	0.071734	0.092068
$u = 20$	1	0.009262	0.021096	0.017957	0.023657
	3	0.013923	0.035457	0.029480	0.040453
	5	0.014553	0.038199	0.031570	0.043769
	∞	0.014652	0.038773	0.031994	0.044477

References

Albrecher, H., Constantinescu, C. and Loisel, S. (2011). Explicit ruin formulas for models with dependence among risks. *Insurance: Mathematics and Economics*, 48, 265–270.

Asmussen, S. (2000). Ruin probabilities. World Scientific, Singapore.

Dickson, D.C.M. (2005). Insurance risk and ruin. Cambridge University Press.

Dickson, D.C.M. and Waters, H.R. (1992). The probability and severity of ruin in finite and infinite time. *Astin Bulletin*, 22(2), 178–190.

Garcia, J.M.A. (2005). Explicit solutions for survival probabilities in the classical risk model. *Astin Bulletin*, 35, 1, 113–130.

Gerber, H.U. (1979). An introduction to mathematical risk theory. Huebner Foundation Monograph 8.

Gerber, H.U, Goovaerts, M.J. and Kaas, R. (1987). On the probability and severity of ruin. *Astin bulletin*, 17(2), 151–163.

Ghitany, M.E., Atieh, B., Nadarajah, S. (2008). Lindley distribution and its application. *Mathematics and Computers in Simulation*, 78, 493–506.

Ghitany, M.E. and Al–Mutairi, D.K. (2009). Estimation methods for the discrete Poisson–Lindley distribution. *Journal of Statistical Computation and Simulation*, 79(1), 1–9.

Gómez–Déniz, E. and Calderín-Ojeda, E. (2011). The discrete Lindley distribution: properties and applications. *Journal of Statistical Computation and Simulation*, 81(11), 1405–1416.

Gómez–Déniz, E., Sarabia, J.M. and Balakrhisnan, N. (2012). A multivariate discrete Poisson-Lindley distribution: extensions and actuarial applications. *Astin Bulletin*, 42(2), 655–678.

Grandell, J. (1990). Aspects of Risk Theory. Springer–Verlag, New York.

Grandell, J. and Segerdahl, C.-O. (1971). A comparison of some approximations of ruin probabilities. *Skand. AktuarTidskr.*, 144–158.

Lindley, D.V. (1958). Fiducial distributions and Bayes's theorem. *Journal of the Royal Statistical Society, Ser. B*, 20(1), 102–107.

Panjer, H.H. and Willmot, G.E. (1992). *Insurance Risk Models.* Society of Actuaries, Schaumburg.

Ramsay, C. M. (2003). A solution to the ruin problem for Pareto distributions. *Insurance: Mathematics and Economics*, 33, 109–116.

Rolski, T., Schmidli, H. Schmidt, V. y Teugel, J. (1999). Stochastic processes for insurance and finance. John Wiley & Sons.

Sankaran, M. (1971). The discrete Poisson–Lindley distribution. *Biometrics*, 26(1), 145–149.

Wei, L. and Yang, H.–l. (2004). Explicit expressions for the ruin probabilities of Erlang risk process with Pareto individual claim. *Acta Mathematicae Applicatae Sinica*, 20(3), 495–506.

Zakerzadeh, H. and Dolati, A. (2009). Generalized Lindley Distribution. *Journal of Mathematical Extension*, 3(2), 13–25.